

Homological algebra
Homework (due November 7th)

You can compose in english or in french. You can freely (no need to make a precise reference) use any result obtained prior to the statement of the exercise in the (online) course.

1. Show that, if k is a commutative ring, then $\mathbf{Op}(k\text{-}\mathbf{Mod})$ is isomorphic to $k[t]\text{-}\mathbf{Mod}$.
2. Show that a split monomorphism is a regular monomorphism and that a regular monomorphism is a monomorphism (and dual).
3. Show that a morphism $X \rightarrow Y$ in a category $\mathcal{C}$ is a monomorphism (resp. an epimorphism) if and only if the induced functor $\mathcal{C}_{/X} \rightarrow \mathcal{C}_{/Y}$ (resp. ${}_Y\mathcal{C} \rightarrow {}_X\mathcal{C}$) is fully faithful.
4. Show that colimits of *sets* are stable under pullback.
5. Show that, if k is a commutative ring, then $k[t]$ (resp. $k[t]_t$) endowed with

$$\mu : t \mapsto t \otimes 1 + 1 \otimes t \quad (\text{resp. } \nu : t \mapsto t \otimes t)$$

is an abelian group of the category opposite to the category of k -algebras.